

# PETROPHYSICAL ROCK PHYSICS MODELING: A COMPARISON OF THE KRIEF AND GASSMANN EQUATIONS, AND APPLICATIONS TO VERIFYING AND ESTIMATING COMPRESSIONAL AND SHEAR VELOCITIES

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## ABSTRACT

The Gassmann Equation, first published in the 1950's, relates compressional acoustic velocity to bulk moduli of the empty rock frame, solids and fluids, density and porosity. When combined with Biot's analysis (1961), compressional and shear velocities can be determined. In order to solve the Gassmann/Biot model, it is necessary to determine bulk modulus of the dry rock. Several empirical equations have been published to achieve this.

Krief (1987) approaches the same rock physics model somewhat differently. In addition to elastic moduli of solids and fluids, the model incorporates shear moduli of the solids and the Biot compressibility constants. Thus, both compressional and shear velocities are available from Krief. Both models allow for predictions of velocity variations as fluids change (gas vs. water).

An initial rock model is formulated, consisting of rock matrix volume, shale volume, and porosity. Best source data for porosity is a density/neutron combination, because calculations are relatively independent of fluid content and matrix lithology.

Both models, adapted to petrophysical applications, involve average bulk moduli for fluid (water and gas) as a function of pressure. Zone input includes average values over the zone depth of:

- Compressional travel time – matrix
- Compressional travel time – shale
- Shear travel time – matrix
- Shear travel time – shale

Both models are a comprehensive solution to velocity properties of rocks over the complete range of lithologies, shale content and porosity.

In high porosity soft rocks (porosity values of 40% or greater) both models give comparable results – compressional velocity slowing in the presence of gas. However, at lower porosities, and particularly for rocks with less than 20% porosity, the Gassmann model predicts a much larger velocity slowing effect than does Krief, depending upon the dry rock bulk modulus model that is applied.

When applying the models, for calibration of any one reservoir, it is best to have both compressional and shear measurements. Then, by suitable adjustment of the zone parameters, a match between the pseudo compressional and shear data (modeled logs) and real measurements can be achieved.

In many wells, and particularly for shear data, occasional depth intervals show emphatic differences between the pseudo and measured data. In such cases assuming that porosity calculations are reliable (no bad hole problems), data processing to generate the shear curve is suspect.

Once a reservoir sequence has been correctly zoned, the pseudo shear curve is reliable, even if no shear measurements have been collected. For key wells where no acoustic measurements of any kind have been collected, pseudo compressional and shear velocity data can be estimated, and used for geophysical calibration.

Examples from a number of clastic reservoirs are presented.

## INTRODUCTION

Acoustic properties of rock (both compressional and shear responses) are influenced by the mechanical properties of the solid components as well as the compressibility of the contained fluids – oil, gas, free

water, bound water. Standard petrophysical interpretation of compressional acoustic data involves empirical relations such as the Wyllie (1956) time series equation and the Raymer-Hunt-Gardner (1980) transform. It is well known that these equations can be suspect particularly if gas is present. Petrophysical interpretation of shear data is even more approximate - usually involving average ratios of compressional to shear travel times for different matrix/shale components.

Because compressional and shear acoustic properties are so important in geophysical seismic interpretation, a great deal of effort has been denoted to understanding velocity behavior for both compressional and shear energy. A number of relationships have been developed based on fundamental rock and fluid properties. Most of these analyses have not been expressed in ways suitable for petrophysical application. In addition there is often inconsistency in terminology and mnemonics, which complicates comparisons among the various models proposed. In this paper, we analyzed some of these relationships, and present equations that can be used for the interpretations of standard wireline log data.

**DEFINATION OF ROCK AND FLUID PROPERTIES**

The following definitions are used in subsequent analyses (synonyms from the literature are indicated)

$$K = \text{Bulk Modulus} = \rho_B \left( V_p^2 - \frac{4}{3} V_s^2 \right)$$

$$M = \text{Elastic Modulus} = V_p^2 \times \rho_B = K + \frac{4}{3} \mu$$

$$\mu = \text{Shear Modulus} = V_s^2 \times \rho_B$$

$$\beta_B = \text{Biot Compressibility Constant} = \phi^{3/1-\phi}$$

$M_B$  = Rock & Fluid Biot Coefficient

$$1/M_B = \frac{\beta_B - \phi}{K_{ma}} + \frac{\phi}{K_F}$$

$V_p = V_c$  = Compressional velocity

$V_s$  = Shear velocity

$\rho_b$  = Bulk density

$\phi$  = Porosity

$K_{sat}$  = Bulk modulus of rock, fluid saturated

$K_{dry}$  = Bulk modulus of dry rock

$K_o = K_{ma}$  = Bulk modulus of the matrix

$K_F = K_{Fluid}$  = Bulk modulus of the fluid

$\mu_{ma}$  = Shear modulus of the matrix

$\mu_{sat}$  = Shear modulus of the saturated rock

$\mu_{dry}$  = Shear modulus of the dry rock

$M_{sat}$  = Elastic modulus of saturated rock

$M_{dry}$  = Elastic modulus of dry rock

$M_o$  = Elastic modulus of rock mineral

$M_F$  = Elastic modulus of the fluid

DTC = DTP = Compressional Travel Time

DTS = Shear Travel Time

**SOURCE EQUATIONS FROM GEOPHYSICAL LITERATURE**

**A. Gassmann/Biot**

$$\frac{K_{sat}}{K_o - K_{sat}} = \frac{K_{dry}}{K_o - K_{dry}} + \frac{K_{Fluid}}{\phi(K_o - K_{Fluid})}$$

The model assumes  $\mu_{sat} = \mu_{dry}$

**B. Simplified Gassmann – published by Crain (1986)**

Gives only  $V_p$  data

$$M_{sat} = M_{dry} + \left\{ \frac{\left( 1 - \frac{K_{dry}}{K_o} \right)^2}{\frac{\phi}{K_{fl}} + \frac{1-\phi}{K_o} - \frac{K_{dry}}{K_o^2}} \right\}$$

**C. Simplified Gassmann – published by Mavko (1988)**

$$\frac{M_{sat}}{M_o - M_{sat}} \approx \frac{M_{dry}}{M_o - M_{dry}} + \frac{M_{Fluid}}{\phi(M_o - M_F)}$$

**D. Krief**

$$V_p^2 = \frac{K + \frac{4}{3}\mu}{\rho_B}$$

$$K = K_{ma} + (1 - \beta_B) + \beta_B^2 M_B$$

$$\mu = \mu_{ma} \times (1 - \beta_B)$$

$$K = \rho_B V_p^2 - \rho_B \frac{4}{3} V_s^2$$

$$\frac{1}{M_B} = \frac{\beta_B - \phi}{K_{ma}} + \frac{\phi}{K_f}$$

$$1 - \beta_B = (1 - \phi)^{\frac{3}{1-\phi}}$$

**EQUATIONS USED IN THIS ANALYSIS**

**A. Gassmann (Crain) – to determine  $V_p$  only**

$$M_{sat} = M_{dry} + \left\{ \frac{\left(1 - \frac{K_{dry}}{K_o}\right)^2}{\frac{\phi}{K_{fl}} + \frac{1-\phi}{K_o} - \frac{K_{dry}}{K_o^2}} \right\}$$

The procedures to solve the equations are:

1. Determine  $K_o$  for the specific formation:

Sandstone  $37.9 * 10^{10}$  dynes/cm<sup>2</sup>

Limestone  $67 * 10^{10}$  dynes/cm<sup>2</sup>

Dolomite  $82 * 10^{10}$  dynes/cm<sup>2</sup>

2. Determine  $K_{fluid}$  for any assumed fluid saturation. Common values are:

Water  $3.05 * 10^{10}$  dynes/cm<sup>2</sup>

Oil  $0.43 * 10^{10}$  dynes/cm<sup>2</sup>

Gas  $0.043 * 10^{10}$  dynes/cm<sup>2</sup>

Values for gas are pressure and component – dependent

3. Determine  $K_{dry}$ . Crain suggests an average value of about  $2 * 10^{10}$  dynes per cm<sup>2</sup>, but

recognizes the range could be from 1.5 to 5 \*  $10^{10}$  dynes per cm<sup>2</sup>.

Several empirical relationships have been suggested to predict  $K_{dry}$  from  $K_o$  and porosity:

$$\text{Geertsma (1961)} \quad \frac{1}{K_{dry}} = \frac{1}{K_o} \times (1 + 50\phi)$$

$$\text{Nur (1995)} \quad K_{dry} = K_o \left(1 - \frac{\phi}{\phi_c}\right)$$

Where  $\phi_c$  is critical porosity

Material	$\phi_c$ %
Sandstone	40
Limestone	60
Dolomite	40
Chalk	65
Igneous	5

$$\text{Biot} \quad K_{dry} = K_o (1 - \phi)^{\frac{3}{1-\phi}}$$

**B. Gassmann/Biot – to determine  $V_p$  and  $V_s$**

$$\frac{K_{sat}}{K_o - K_{sat}} \approx \frac{K_{dry}}{K_o - K_{dry}} + \frac{K_{fl}}{\phi(K_o - K_{fl})}$$

To extract  $V_p$  and  $V_s$  from  $K$  it is necessary to determine  $\mu$

$\mu_{sat} = \mu_{ma} (1 - \beta)$ ;  $\mu_{ma}$  is available from rock mechanics data

Sandstone  $44 * 10^{10}$  dynes/cm<sup>2</sup>

Limestone  $32 * 10^{10}$  dynes/cm<sup>2</sup>

Dolomite  $54 * 10^{10}$  dynes/cm<sup>2</sup>

**C. Krief – to determine  $V_p$  and  $V_s$**

$$V_p^2 = \frac{K + \frac{4}{3}\mu}{\rho_B}$$

$$K = K_{ma} + (1 - \beta_B) + \beta_B^2 M_B$$

$$\mu = \mu_{ma} \times (1 - \beta_B)$$

$$K = \rho_B V_p^2 - \rho_B \frac{4}{3} V_s^2$$

$$\frac{1}{M_B} = \frac{\beta_B - \phi}{K_{ma}} + \frac{\phi}{K_f}$$

$$1 - \beta_B = (1 - \phi)^{\frac{3}{1-\phi}}$$

Input parameters are similar to the Gassmann/Biot equation except there is no requirement to solve for  $K_{dry}$ .

**METHODOLOGY**

All of the equations require determination of the following parameters:

$K_o = K_{ma}$  = Bulk modulus of the matrix

$\rho_B$  = Bulk density

$K_{Fluid} = K_F$  = Bulk modulus of the fluid combination

$\mu$  = Shear modulus of the rocks – Gassmann/Biot and Krief models

$K_{dry}$  = Bulk modulus of dry rocks – Gassmann (Crain and Gassmann/Biot)

$\phi$  = Porosity

In our approach we incorporate the influence of shale by involving bulk moduli of the shale component.

The methodology consists of:

1. Determine, at each level, a rock model of:

Matrix volume

Shale volume

Porosity

This is best achieved using density/neutron combination, because calculations are relatively independent of fluid content and matrix lithology.

2. Assuming different fluid combination (gas vs. water) calculate:

$\rho_B$

$V_p$

$V_s$

Neutron log response

3. From  $V_p$  and  $V_s$ , determine DTC and DTS

4. Compare calculated values with log data. The comparison, level-by-level, will indicate what fluid combination is “seen” by each log.

The methodology allows the calculation of acoustic properties (both compressional and shear) even if no acoustic log data has been run on the well.

**Models to Establish  $K_{dry}$  in the Gassmann Equation**

The four models which can be used to estimate  $K_{dry}$  from  $K_o$  are:

1. Crain – Infers a constant value of  $K_{dry}$  of  $1.86 * 10^{10}$  dynes/cm<sup>2</sup>
2. Geertsma
3. Biot
4. Nur

Models 2, 3, and 4 give quite different results as expressed in the following table ( $K_{dry}$  in dynes/cm \*  $10^{10}$ )

Porosity	K <sub>dry</sub> Geertsma	K <sub>dry</sub> Biot	K <sub>dry</sub> Nur
5	10.8	32.3	33.2
10	6.3	26.7	28.5
15	4.47	21.4	23.7
20	3.5	16.5	19.0
25	2.80	12.0	14.4
30	2.37	8.5	9.5
35	2.0	5.2	4.75

The implications of the differences are very significant. Velocity slowing due to the presence of gas are predicted to be much greater if the Crain or Geertsma models are used as compared with the Biot or Nur models. When compared with Krief (which does not involve K<sub>dry</sub>) a match would be achieved if a model close to Nur is applied.

**EXAMPLES**

Data from the following reservoirs are presented:

1. High porosity shallow sandstone from SW Wyoming
2. High porosity sandstone from the Gulf of Mexico
3. Low porosity tight gas sandstone from NW Colorado
4. Low porosity tight gas sandstone from Central Wyoming

For each example, the following plots are shown:

Reconstructed porosity logs (including compressional and shear acoustic data), compared with actual wireline logs for the following models:

- Gassmann using Geertsma model to calculate K<sub>0</sub>
- Gassmann using Biot model to calculate K<sub>0</sub>
- Krief

**DISCUSSION OF RESULTS**

**1. High Porosity Gas Sandstone, SW Wyoming**

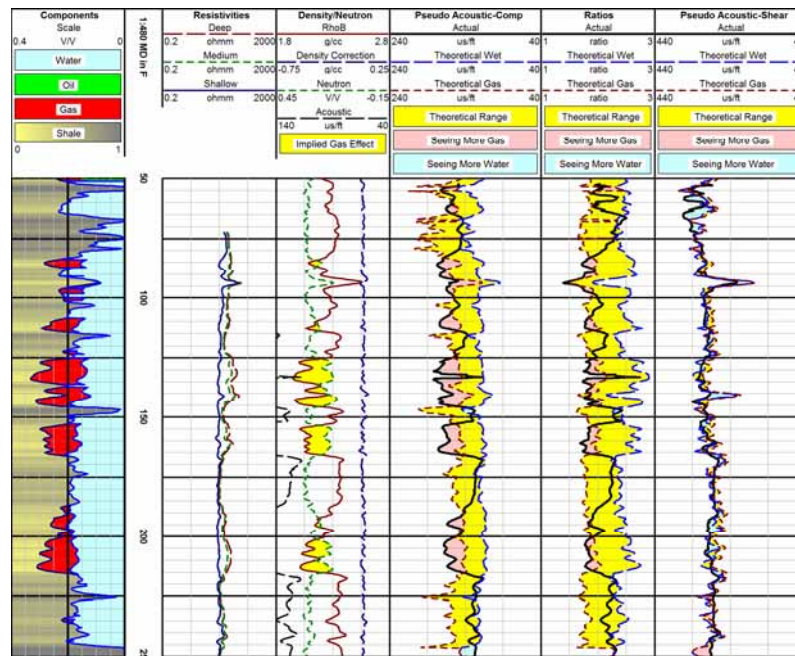


Fig 1: Gassmann (Geertsma Model) – Difference between theoretical wet DTC and Gas filled DTC is about 20 – 30 microseconds per foot. Actual DTC meanders between theoretical wet and theoretical gas. Generally, when gas is

indicated from density/neutron cross over, the actual DTC is close to the gas theoretical curve. Reconstruction of the theoretical DTS agrees well with measured DTS.

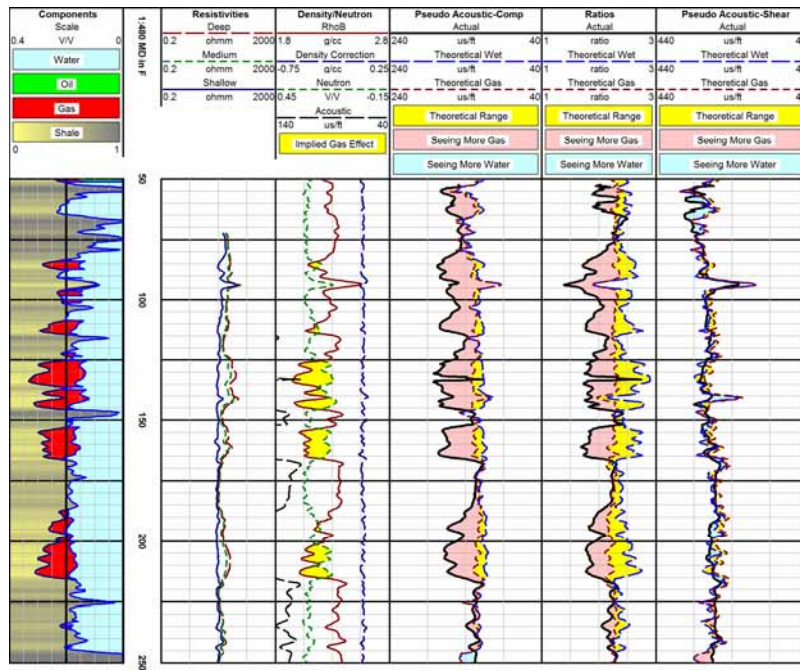


Fig 2: Gassmann (Biot Model) – There is very little difference between theoretical DTC gas and theoretical DTC wet and the theoretical curves do not match measured DTC. Theoretical DTS is a close match to measured DTS.

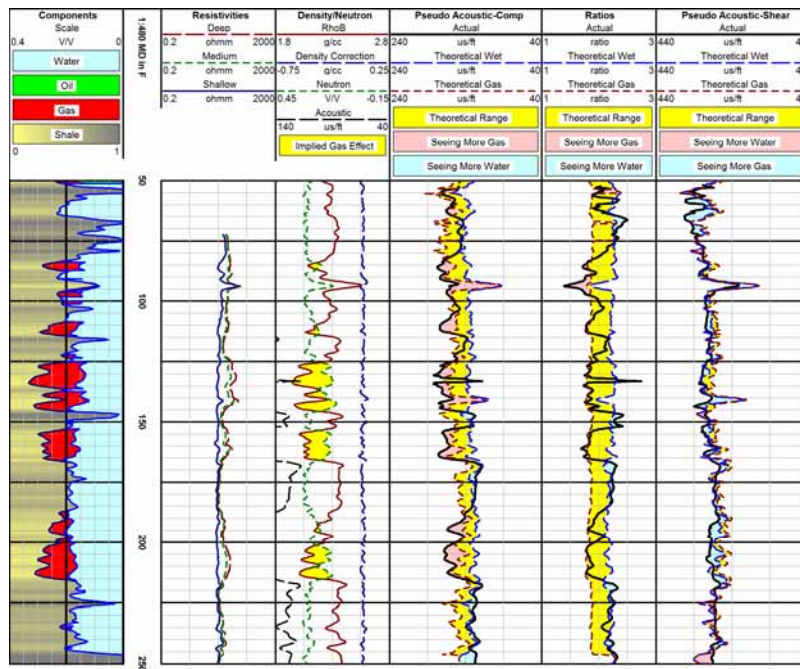


Fig 3: Krief – Theoretical DTC gas is about 20 microseconds per foot higher than theoretical DTC wet. Actual DTC meanders between theoretical DTC gas and theoretical DTC wet in a manner consistent with gas indications from density/neutron response. Reconstructed DTS agrees well with measured DTS.



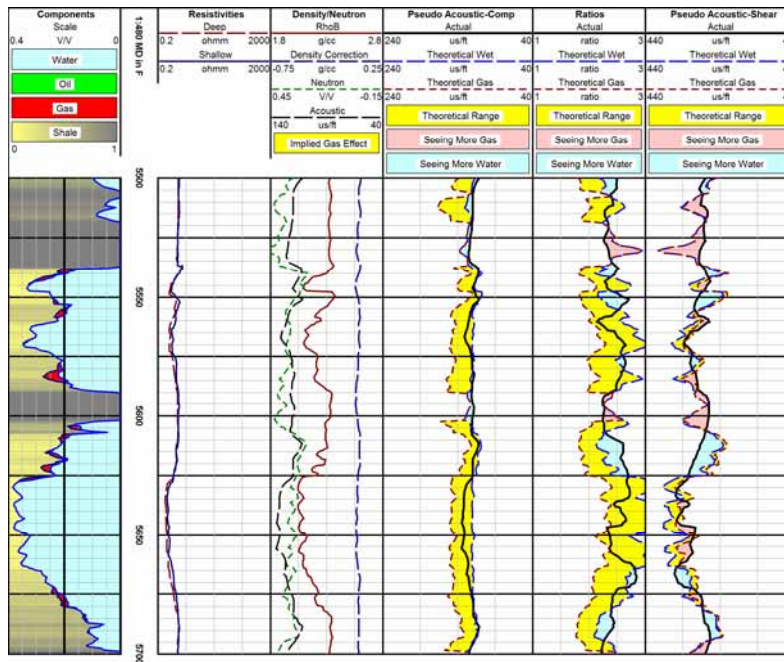


Fig 5: Gassmann (Biot Model) – Separation between theoretical DTC gas and DTC wet is about 20 microseconds per foot. The actual DTC tracks the theoretical wet DTC (as it should). There is fair agreement between actual and theoretical DTS, and the theoretical DTS ratio (wet) mostly tracks the actual DTS ratio quite well.

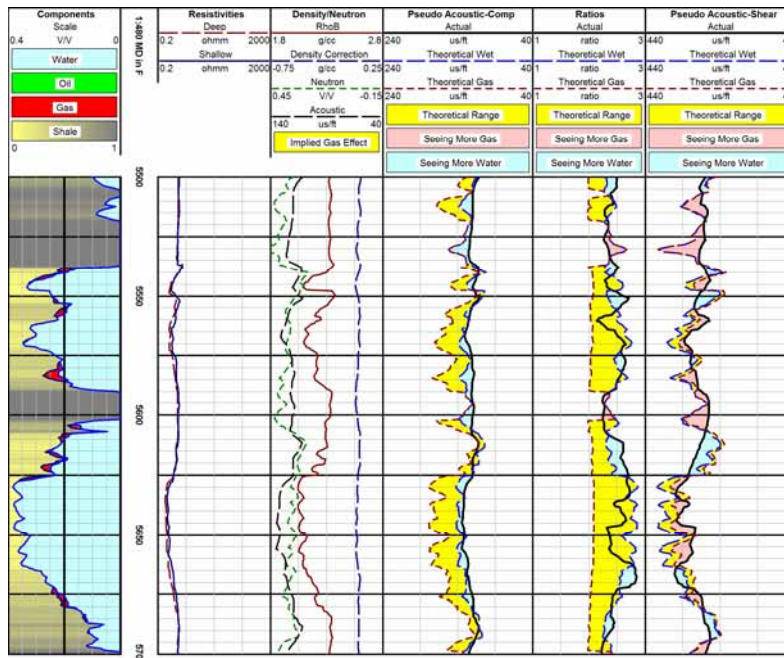


Fig 6: Krief – Separation between theoretical DTC gas and DTC wet is about 30 microseconds per foot. Actual DTC tracks theoretical DTC wet more closely than for the Gassmann models. Agreement between theoretical and actual DTS is a little better than for the Gassmann models. There is good agreement between actual ratio and theoretical wet ratio.



3. Low Porosity, Tight Gas Sandstone from NW Colorado

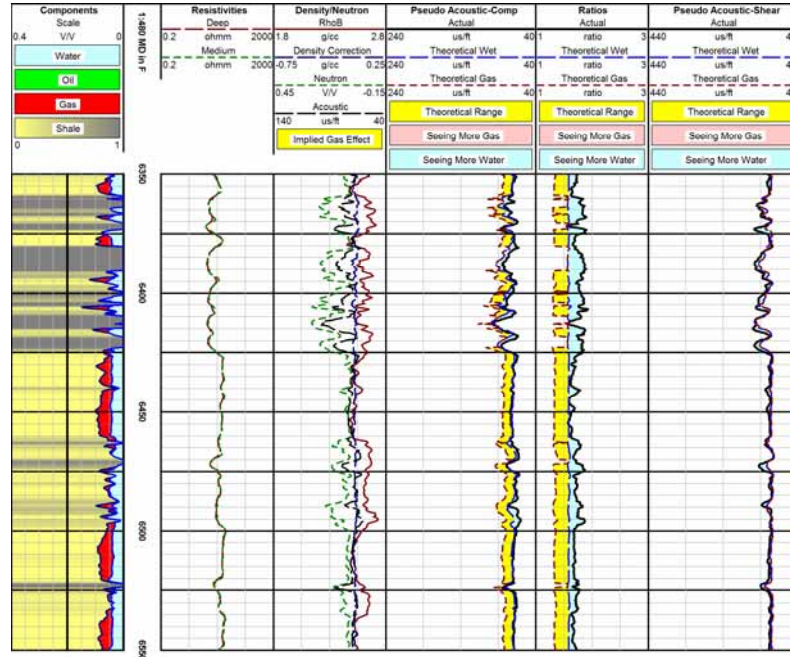


Fig 7: Gassmann (Geertsma Model) – Separation between theoretical DTC gas and DTC wet is about 10 microseconds per foot. Theoretical DTS agrees well with measured DTS, but the ratios of the two theoretical curves do not match the measured ratio.

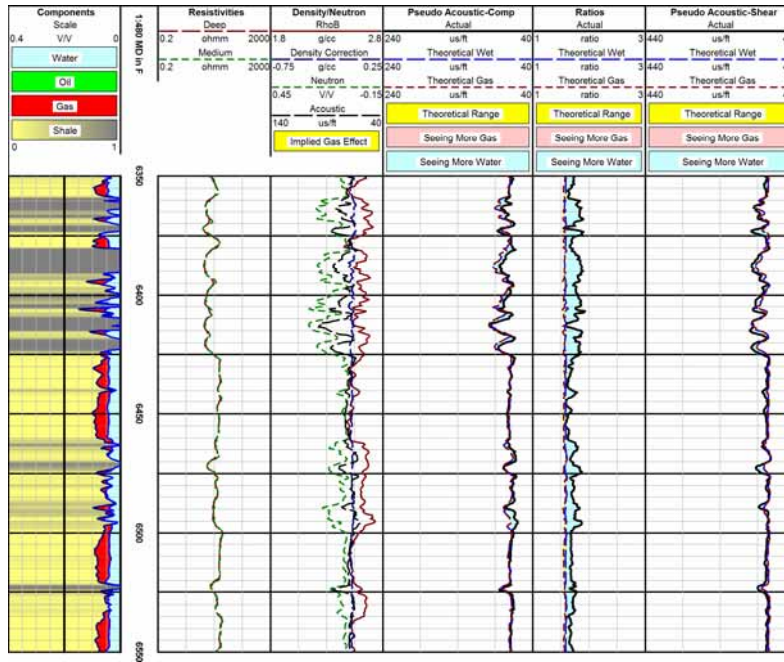


Fig 8: Gassmann (Biot Model) – There is no separation between theoretical DTC gas and DTC wet, but the match with measured DTC is excellent. Theoretical DTS agrees quite well with measured DTS, but the theoretical ratio curve does not match measured ratio.

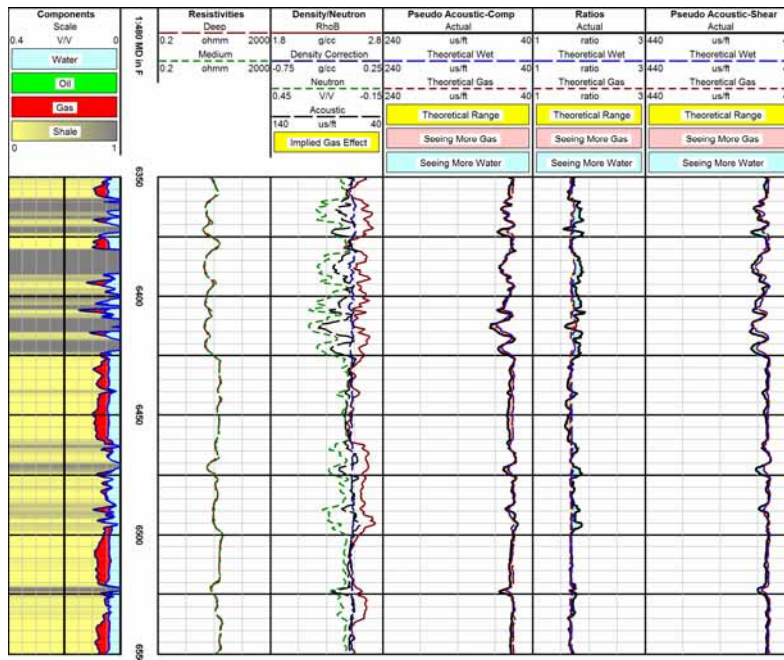


Fig 9: Krief – There is no separation between theoretical DTC gas and DTC wet. However, all theoretical curves (including ratios) agree well with measured data.

4. Tight Gas Sandstone, Powder River Basin, Wyoming

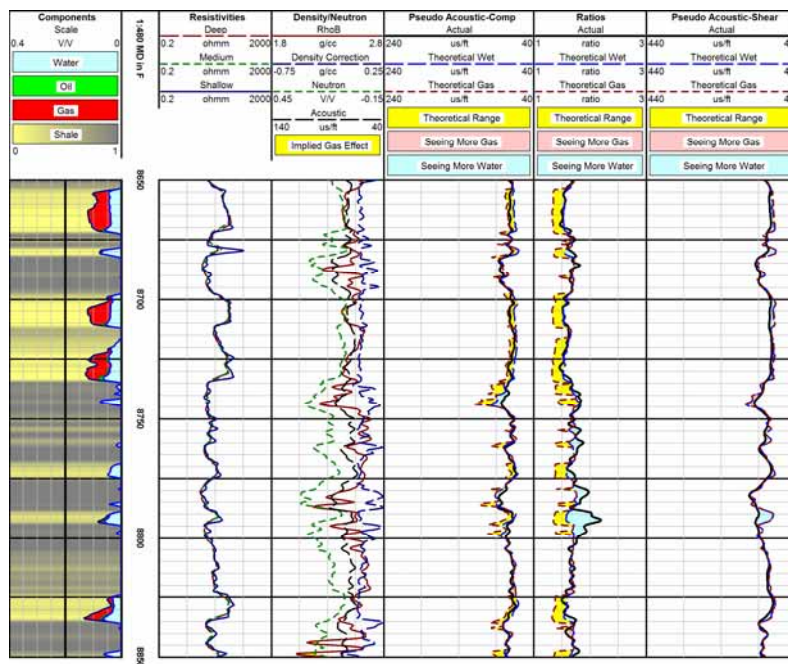


Fig 10: Gassmann (Geertsma Model) – Separation between theoretical DTC gas and DTC wet is about 10 microseconds per foot. Theoretical DTS agrees well with actual DTS. Theoretical wet ratio of DTS to DTC agrees well with measured DTS to DTC ratio.

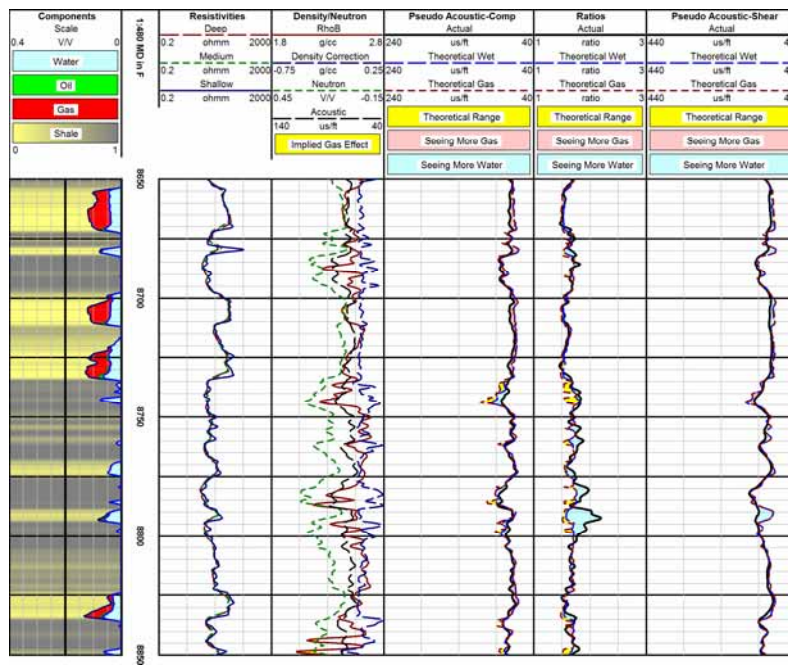


Fig 11: Gassmann (Biot Model) – There is no separation between theoretical DTC gas and DTC wet. All theoretical curves agree well with measured curves.

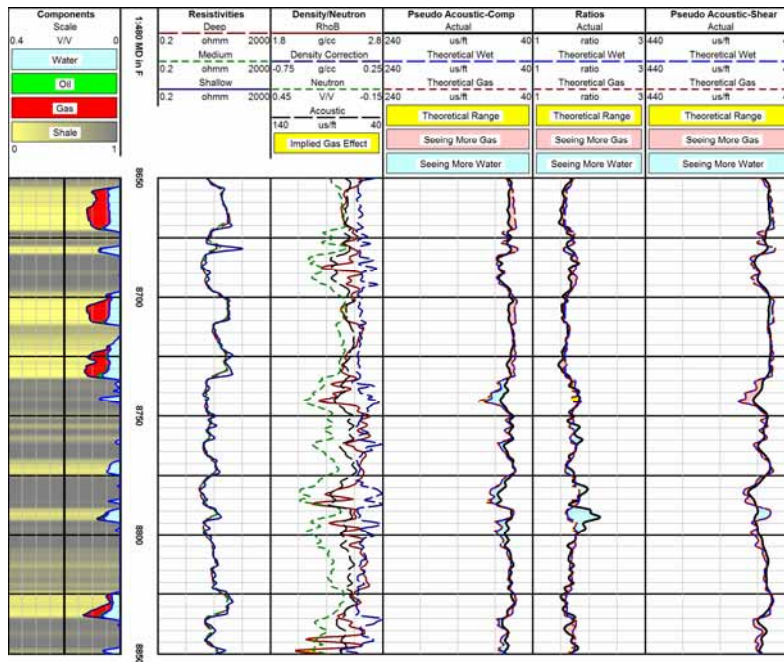


Fig 12: Krief – There is no separation between theoretical DTC gas and DTC wet. All theoretical curves agree well with measured data.

**VELOCITY SLOWING DUE TO THE PRESENCE OF GAS**

The three different Gassman K Dry models presented show different predictions with respect to the values of K Dry as porosity dependent functions (K Dry is in dynes per cm<sup>2</sup> x 10<sup>10</sup>). As a consequence, each model gives different results with respect to velocity slowing in the presence of gas.

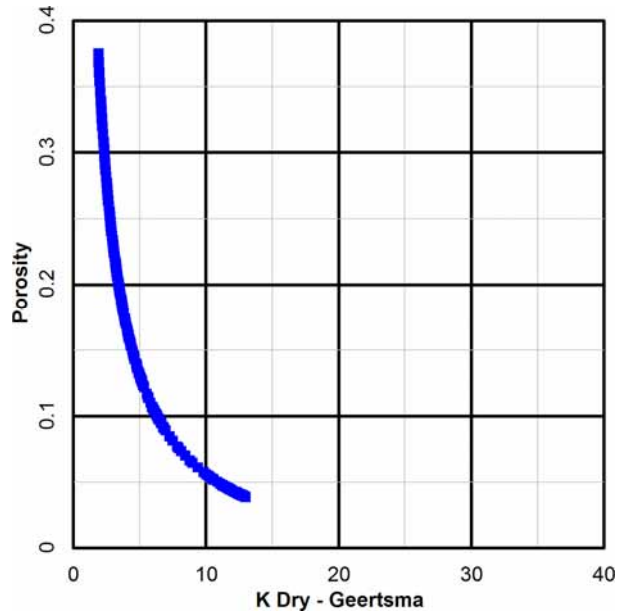


Fig 13: K Dry Geertsma vs. Porosity

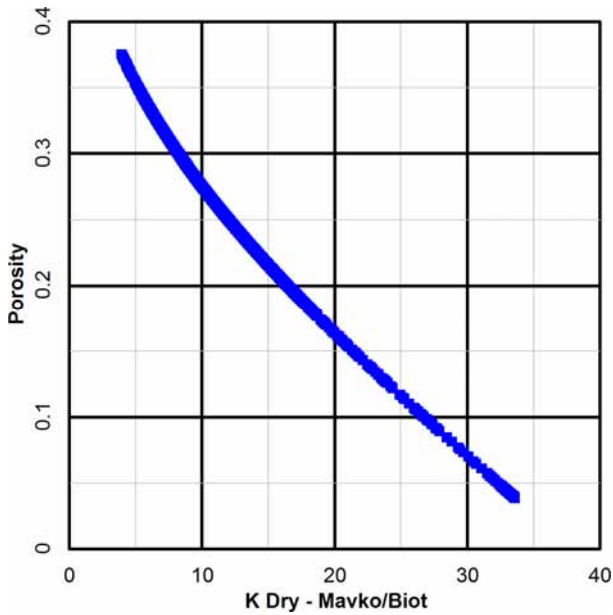


Fig 14: K Dry Mavko/Biot vs. Porosity

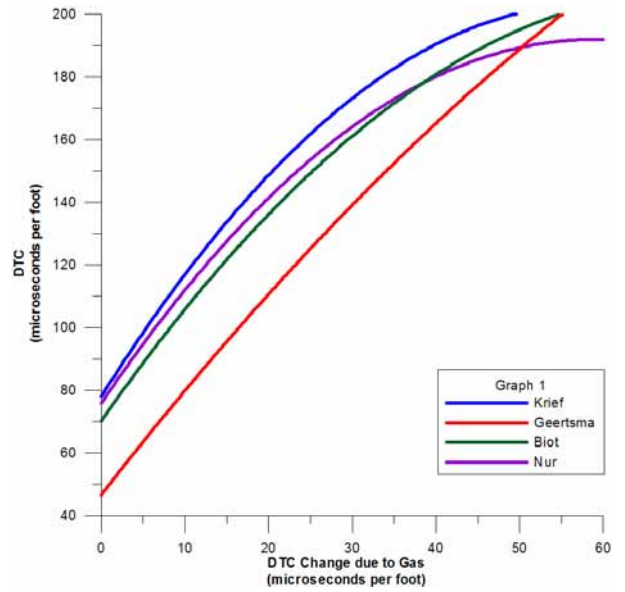


Fig 16: Comparisons of the models

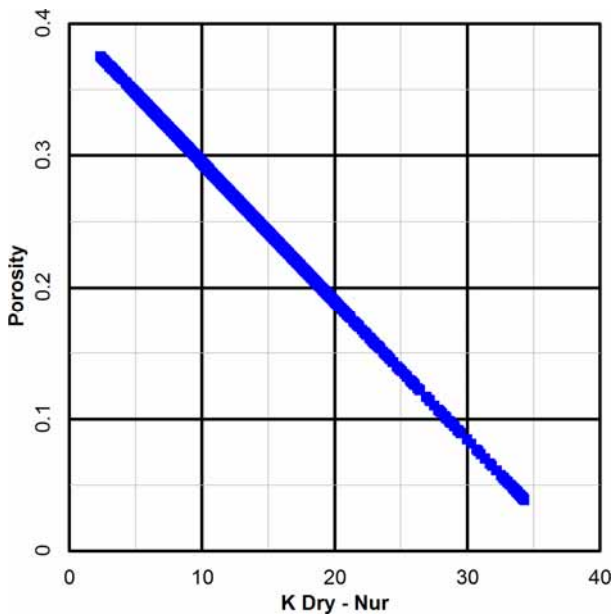


Fig 15: K Dry Nur vs. Porosity

Figure 16 summarizes the results; data are based on an assumed gas saturation of 80%. The conclusion is reached that the Geertsma dry rock bulk modulus model leads to significant overestimates of the effect of gas, particularly in porosities less than about 20% (DT of 85 microseconds per foot).

## CONCLUSIONS

1. Petrophysical adaptation of the Gassmann and Krief acoustic models has been successfully applied to the analysis of acoustic compressional and shear data. Good comparisons exist with measured data.
2. For the Gassmann model, it is necessary to estimate dry rock bulk modulus. A number of published empirical equations were used. It appears that the Biot and Nur models give better and more consistent results over the entire porosity range than does the Geertsma model; the Geertsma model leads to significant overestimation as to the degree of velocity slowing (increase in DT) in the presence of gas.
3. The Krief model, which does not involve dry rock bulk modulus, appears to be stable for the entire range of porosity. It is relatively close to the Gassmann (Biot and Nur) models, but somewhat more consistent.
4. From either the Gassmann/Biot or Krief models, it appears that velocity slowing due to the presence of gas is no more than three microseconds per foot in rocks with less than about 15% porosity.
5. The techniques presented allow for the prediction of acoustic properties (both compressional and shear) from other standard wireline log measurements. Thus, pseudo compressional and shear data can be reliably estimated curve when such measurements have not been made. It is necessary to calibrate from wells in the area, since accurate values of matrix and shale properties are required.

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